Appendix C — Numerical Data

$$\boldsymbol{D}^1\operatorname{slog}_{\boldsymbol{e}}(0)_{40} = 0.915944781172534 \qquad \boldsymbol{D}^2\operatorname{slog}_{\boldsymbol{e}}(0)_{40} = 0.498696444588079$$
$$\boldsymbol{D}^1\operatorname{slog}_{\boldsymbol{e}}(0)_{50} = 0.915945263266249 \qquad \boldsymbol{D}^2\operatorname{slog}_{\boldsymbol{e}}(0)_{50} = 0.498704465665853$$
$$\boldsymbol{D}^1\operatorname{slog}_{\boldsymbol{e}}(0)_{60} = 0.915945536274640 \qquad \boldsymbol{D}^2\operatorname{slog}_{\boldsymbol{e}}(0)_{60} = 0.498707053886320$$
$$\boldsymbol{D}^1\operatorname{slog}_{\boldsymbol{e}}(0)_{70} = 0.915945731075678 \qquad \boldsymbol{D}^2\operatorname{slog}_{\boldsymbol{e}}(0)_{70} = 0.498708123849263$$
$$\boldsymbol{D}^1\operatorname{slog}_{\boldsymbol{e}}(0)_{80} = 0.915945846988776 \qquad \boldsymbol{D}^2\operatorname{slog}_{\boldsymbol{e}}(0)_{80} = 0.498708709807559$$
$$\boldsymbol{D}^1\operatorname{slog}_{\boldsymbol{e}}(0)_{90} = 0.915945908914126 \qquad \boldsymbol{D}^2\operatorname{slog}_{\boldsymbol{e}}(0)_{90} = 0.498708986251720$$
$$\boldsymbol{D}^1\operatorname{slog}_{\boldsymbol{e}}(0)_{100} = 0.915945951095597 \qquad \boldsymbol{D}^2\operatorname{slog}_{\boldsymbol{e}}(0)_{100} = 0.498709096804952$$
$$\boldsymbol{D}^1\operatorname{slog}_{\boldsymbol{e}}(0)_{110} = 0.915945982447644 \qquad \boldsymbol{D}^2\operatorname{slog}_{\boldsymbol{e}}(0)_{110} = 0.498709165188107$$
$$\boldsymbol{D}^1\operatorname{slog}_{\boldsymbol{e}}(0)_{120} = 0.915946001992852 \qquad \boldsymbol{D}^2\operatorname{slog}_{\boldsymbol{e}}(0)_{120} = 0.498709210043943$$
$$\boldsymbol{D}^1\operatorname{slog}_{\boldsymbol{e}}(0)_{130} = 0.915946014757482 \qquad \boldsymbol{D}^2\operatorname{slog}_{\boldsymbol{e}}(0)_{130} = 0.498709225011490$$
$$\boldsymbol{D}^1\operatorname{slog}_{\boldsymbol{e}}(0)_{140} = 0.915946025105883 \qquad \boldsymbol{D}^2\operatorname{slog}_{\boldsymbol{e}}(0)_{140} = 0.498709229740035$$
$$\boldsymbol{D}^1\operatorname{slog}_{\boldsymbol{e}}(0)_{150} = 0.915946032676321 \qquad \boldsymbol{D}^2\operatorname{slog}_{\boldsymbol{e}}(0)_{150} = 0.498709235488693$$
$$\boldsymbol{D}^1\operatorname{slog}_{\boldsymbol{e}}(0) \approx 0.91594603 \qquad \boldsymbol{D}^2\operatorname{slog}_{\boldsymbol{e}}(0) \approx 0.49870923$$

$$\boldsymbol{D}^3\operatorname{slog}_{\boldsymbol{e}}(0)_{40} = -0.66276507305193 \qquad \boldsymbol{D}^4\operatorname{slog}_{\boldsymbol{e}}(0)_{40} = -2.25390534517765$$
$$\boldsymbol{D}^3\operatorname{slog}_{\boldsymbol{e}}(0)_{50} = -0.66276873212276 \qquad \boldsymbol{D}^4\operatorname{slog}_{\boldsymbol{e}}(0)_{50} = -2.25423131435349$$
$$\boldsymbol{D}^3\operatorname{slog}_{\boldsymbol{e}}(0)_{60} = -0.66277398975381 \qquad \boldsymbol{D}^4\operatorname{slog}_{\boldsymbol{e}}(0)_{60} = -2.25434721926022$$
$$\boldsymbol{D}^3\operatorname{slog}_{\boldsymbol{e}}(0)_{70} = -0.66277900884800 \qquad \boldsymbol{D}^4\operatorname{slog}_{\boldsymbol{e}}(0)_{70} = -2.25440263020639$$
$$\boldsymbol{D}^3\operatorname{slog}_{\boldsymbol{e}}(0)_{80} = -0.66278207800690 \qquad \boldsymbol{D}^4\operatorname{slog}_{\boldsymbol{e}}(0)_{80} = -2.25443382098240$$
$$\boldsymbol{D}^3\operatorname{slog}_{\boldsymbol{e}}(0)_{90} = -0.66278377739784 \qquad \boldsymbol{D}^4\operatorname{slog}_{\boldsymbol{e}}(0)_{90} = -2.25444919899063$$
$$\boldsymbol{D}^3\operatorname{slog}_{\boldsymbol{e}}(0)_{100} = -0.66278506194145 \qquad \boldsymbol{D}^4\operatorname{slog}_{\boldsymbol{e}}(0)_{100} = -2.25445694299939$$
$$\boldsymbol{D}^3\operatorname{slog}_{\boldsymbol{e}}(0)_{110} = -0.66278603921500 \qquad \boldsymbol{D}^4\operatorname{slog}_{\boldsymbol{e}}(0)_{110} = -2.25446221464863$$
$$\boldsymbol{D}^3\operatorname{slog}_{\boldsymbol{e}}(0)_{120} = -0.66278664482280 \qquad \boldsymbol{D}^4\operatorname{slog}_{\boldsymbol{e}}(0)_{120} = -2.25446557924414$$
$$\boldsymbol{D}^3\operatorname{slog}_{\boldsymbol{e}}(0)_{130} = -0.66278706373557 \qquad \boldsymbol{D}^4\operatorname{slog}_{\boldsymbol{e}}(0)_{130} = -2.25446727337134$$
$$\boldsymbol{D}^3\operatorname{slog}_{\boldsymbol{e}}(0)_{140} = -0.66278741544855 \qquad \boldsymbol{D}^4\operatorname{slog}_{\boldsymbol{e}}(0)_{140} = -2.25446838669198$$
$$\boldsymbol{D}^3\operatorname{slog}_{\boldsymbol{e}}(0)_{150} = -0.66278766900692 \qquad \boldsymbol{D}^4\operatorname{slog}_{\boldsymbol{e}}(0)_{150} = -2.25446928157285$$
$$\boldsymbol{D}^3\operatorname{slog}_{\boldsymbol{e}}(0) \approx -0.662787 \qquad \boldsymbol{D}^4\operatorname{slog}_{\boldsymbol{e}}(0) \approx -2.25446$$

$$\boldsymbol{D}^5 \operatorname{slog}_{\boldsymbol{e}}(0)_{40} = 1.20123676054844 \qquad \boldsymbol{D}^6 \operatorname{slog}_{\boldsymbol{e}}(0)_{40} = 25.8189589561658$$
$$\boldsymbol{D}^5 \operatorname{slog}_{\boldsymbol{e}}(0)_{50} = 1.20028806575165 \qquad \boldsymbol{D}^6 \operatorname{slog}_{\boldsymbol{e}}(0)_{50} = 25.8329970026009$$
$$\boldsymbol{D}^5 \operatorname{slog}_{\boldsymbol{e}}(0)_{60} = 1.20013405434286 \qquad \boldsymbol{D}^6 \operatorname{slog}_{\boldsymbol{e}}(0)_{60} = 25.8387294460917$$
$$\boldsymbol{D}^5 \operatorname{slog}_{\boldsymbol{e}}(0)_{70} = 1.20017665549235 \qquad \boldsymbol{D}^6 \operatorname{slog}_{\boldsymbol{e}}(0)_{70} = 25.8419419969967$$
$$\boldsymbol{D}^5 \operatorname{slog}_{\boldsymbol{e}}(0)_{80} = 1.20021193383909 \qquad \boldsymbol{D}^6 \operatorname{slog}_{\boldsymbol{e}}(0)_{80} = 25.8437963173890$$
$$\boldsymbol{D}^5 \operatorname{slog}_{\boldsymbol{e}}(0)_{90} = 1.20023796411121 \qquad \boldsymbol{D}^6 \operatorname{slog}_{\boldsymbol{e}}(0)_{90} = 25.8447456907652$$
$$\boldsymbol{D}^5 \operatorname{slog}_{\boldsymbol{e}}(0)_{100} = 1.20027097420290 \qquad \boldsymbol{D}^6 \operatorname{slog}_{\boldsymbol{e}}(0)_{100} = 25.8453047049794$$
$$\boldsymbol{D}^5 \operatorname{slog}_{\boldsymbol{e}}(0)_{110} = 1.20029821853748 \qquad \boldsymbol{D}^6 \operatorname{slog}_{\boldsymbol{e}}(0)_{110} = 25.8457046662579$$
$$\boldsymbol{D}^5 \operatorname{slog}_{\boldsymbol{e}}(0)_{120} = 1.20031476521818 \qquad \boldsymbol{D}^6 \operatorname{slog}_{\boldsymbol{e}}(0)_{120} = 25.8459565204653$$
$$\boldsymbol{D}^5 \operatorname{slog}_{\boldsymbol{e}}(0)_{130} = 1.20032838768356 \qquad \boldsymbol{D}^6 \operatorname{slog}_{\boldsymbol{e}}(0)_{130} = 25.8461048506513$$
$$\boldsymbol{D}^5 \operatorname{slog}_{\boldsymbol{e}}(0)_{140} = 1.20034088728237 \qquad \boldsymbol{D}^6 \operatorname{slog}_{\boldsymbol{e}}(0)_{140} = 25.8462167550629$$
$$\boldsymbol{D}^5 \operatorname{slog}_{\boldsymbol{e}}(0)_{150} = 1.20034958138708 \qquad \boldsymbol{D}^6 \operatorname{slog}_{\boldsymbol{e}}(0)_{150} = 25.8463012017336$$
$$\boldsymbol{D}^5 \operatorname{slog}_{\boldsymbol{e}}(0) \approx 1.20034 \qquad \boldsymbol{D}^6 \operatorname{slog}_{\boldsymbol{e}}(0) \approx 25.846$$

$$\boldsymbol{D}^7 \operatorname{slog}_{\boldsymbol{e}}(0)_{40} = 32.9490742321220 \qquad \boldsymbol{D}^8 \operatorname{slog}_{\boldsymbol{e}}(0)_{40} = -495.05095328987$$
$$\boldsymbol{D}^7 \operatorname{slog}_{\boldsymbol{e}}(0)_{50} = 33.0744463270470 \qquad \boldsymbol{D}^8 \operatorname{slog}_{\boldsymbol{e}}(0)_{50} = -495.47688098167$$
$$\boldsymbol{D}^7 \operatorname{slog}_{\boldsymbol{e}}(0)_{60} = 33.1106415509182 \qquad \boldsymbol{D}^8 \operatorname{slog}_{\boldsymbol{e}}(0)_{60} = -495.72547010186$$
$$\boldsymbol{D}^7 \operatorname{slog}_{\boldsymbol{e}}(0)_{70} = 33.1226292184246 \qquad \boldsymbol{D}^8 \operatorname{slog}_{\boldsymbol{e}}(0)_{70} = -495.90651130833$$
$$\boldsymbol{D}^7 \operatorname{slog}_{\boldsymbol{e}}(0)_{80} = 33.1288636927751 \qquad \boldsymbol{D}^8 \operatorname{slog}_{\boldsymbol{e}}(0)_{80} = -496.01445390533$$
$$\boldsymbol{D}^7 \operatorname{slog}_{\boldsymbol{e}}(0)_{90} = 33.1315430903712 \qquad \boldsymbol{D}^8 \operatorname{slog}_{\boldsymbol{e}}(0)_{90} = -496.07230375701$$
$$\boldsymbol{D}^7 \operatorname{slog}_{\boldsymbol{e}}(0)_{100} = 33.1319812761822 \qquad \boldsymbol{D}^8 \operatorname{slog}_{\boldsymbol{e}}(0)_{100} = -496.11212255466$$
$$\boldsymbol{D}^7 \operatorname{slog}_{\boldsymbol{e}}(0)_{110} = 33.1320611530065 \qquad \boldsymbol{D}^8 \operatorname{slog}_{\boldsymbol{e}}(0)_{110} = -496.14179306528$$
$$\boldsymbol{D}^7 \operatorname{slog}_{\boldsymbol{e}}(0)_{120} = 33.1321507609728 \qquad \boldsymbol{D}^8 \operatorname{slog}_{\boldsymbol{e}}(0)_{120} = -496.16027838058$$
$$\boldsymbol{D}^7 \operatorname{slog}_{\boldsymbol{e}}(0)_{130} = 33.1319536106881 \qquad \boldsymbol{D}^8 \operatorname{slog}_{\boldsymbol{e}}(0)_{130} = -496.17242819312$$
$$\boldsymbol{D}^7 \operatorname{slog}_{\boldsymbol{e}}(0)_{140} = 33.1316616124526 \qquad \boldsymbol{D}^8 \operatorname{slog}_{\boldsymbol{e}}(0)_{140} = -496.18231831220$$
$$\boldsymbol{D}^7 \operatorname{slog}_{\boldsymbol{e}}(0)_{150} = 33.1314888709522 \qquad \boldsymbol{D}^8 \operatorname{slog}_{\boldsymbol{e}}(0)_{150} = -496.18954127850$$
$$\boldsymbol{D}^7 \operatorname{slog}_{\boldsymbol{e}}(0) \approx 33.131 \qquad \boldsymbol{D}^8 \operatorname{slog}_{\boldsymbol{e}}(0) \approx -496.18$$

$$\begin{aligned}
\operatorname{slog}_2^{-1}(0.5)_2 &= 1.458961693832438 & \operatorname{slog}_e^{-1}(0.5)_{10} &= 1.6464556716360208\\
\operatorname{slog}_2^{-1}(0.5)_4 &= 1.458655904880133 & \operatorname{slog}_e^{-1}(0.5)_{20} &= 1.6463676325953218\\
\operatorname{slog}_2^{-1}(0.5)_6 &= 1.458692450371729 & \operatorname{slog}_e^{-1}(0.5)_{30} &= 1.6463577769479243\\
\operatorname{slog}_2^{-1}(0.5)_8 &= 1.458741984415978 & \operatorname{slog}_e^{-1}(0.5)_{40} &= 1.6463553806741427\\
\operatorname{slog}_2^{-1}(0.5)_{10} &= 1.458768532260076 & \operatorname{slog}_e^{-1}(0.5)_{50} &= 1.6463546427466649\\
{}^{0.5}2 &\approx 1.4587 & {}^{0.5}e &\approx 1.64635
\end{aligned}$$

$$\begin{aligned}
\operatorname{slog}_e^{-1}(e)_{10} &= 2078.198719173609 & \operatorname{slog}_e^{-1}(\pi)_{10} &= 37105406757.56952\\
\operatorname{slog}_e^{-1}(e)_{20} &= 2076.129166296636 & \operatorname{slog}_e^{-1}(\pi)_{20} &= 37155268624.63599\\
\operatorname{slog}_e^{-1}(e)_{30} &= 2075.998583292668 & \operatorname{slog}_e^{-1}(\pi)_{30} &= 37152290690.85273\\
\operatorname{slog}_e^{-1}(e)_{40} &= 2075.975284589419 & \operatorname{slog}_e^{-1}(\pi)_{40} &= 37150849430.35024\\
\operatorname{slog}_e^{-1}(e)_{50} &= 2075.968983446195 & \operatorname{slog}_e^{-1}(\pi)_{50} &= 37150331380.03964\\
\operatorname{slog}_e^{-1}(e)_{60} &= 2075.967658498696 & \operatorname{slog}_e^{-1}(\pi)_{60} &= 37150112554.57623\\
\operatorname{slog}_e^{-1}(e)_{70} &= 2075.967604923759 & \operatorname{slog}_e^{-1}(\pi)_{70} &= 37149986051.50005\\
\operatorname{slog}_e^{-1}(e)_{80} &= 2075.967631271361 & \operatorname{slog}_e^{-1}(\pi)_{80} &= 37149912712.49439\\
\operatorname{slog}_e^{-1}(e)_{90} &= 2075.967687365403 & \operatorname{slog}_e^{-1}(\pi)_{90} &= 37149874928.20818\\
\operatorname{slog}_e^{-1}(e)_{100} &= 2075.967814831020 & \operatorname{slog}_e^{-1}(\pi)_{100} &= 37149852157.21806\\
\operatorname{slog}_e^{-1}(e)_{110} &= 2075.967925399709 & \operatorname{slog}_e^{-1}(\pi)_{110} &= 37149835758.34629\\
\operatorname{slog}_e^{-1}(e)_{120} &= 2075.967991775568 & \operatorname{slog}_e^{-1}(\pi)_{120} &= 37149825450.22689\\
\operatorname{slog}_e^{-1}(e)_{130} &= 2075.968051571147 & \operatorname{slog}_e^{-1}(\pi)_{130} &= 37149819264.78736\\
\operatorname{slog}_e^{-1}(e)_{140} &= 2075.968108549399 & \operatorname{slog}_e^{-1}(\pi)_{140} &= 37149814532.74459\\
\operatorname{slog}_e^{-1}(e)_{150} &= 2075.968147604069 & \operatorname{slog}_e^{-1}(\pi)_{150} &= 37149810983.64210\\
{}^{e}e &\approx 2075.9681 & {}^{\pi}e &\approx 3.714981\times10^{10}
\end{aligned}$$

Appendix D — Identities

$${}^{-1}x = 0 \qquad {}^{0}x = 1 \qquad {}^{1}x = x \qquad {}^{2}x = x^x$$

$${}^{y}x = \exp_x^y(1)$$
$$\exp_x^y(a) = {}^{y+\operatorname{slog}_x(a)}x$$

$$\operatorname{srt}_2(x) = 1/{}^{\infty}(x^{-1}) = \frac{\log(x)}{W(\log(x))}$$
$$\operatorname{srt}_\infty(x) = 1/{}^{2}(x^{-1}) = \sqrt[x]{x}$$

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